İZMİR UNIVERSITY OF ECONOMICS

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
APPLIED MATHEMATICS AND STATISTICS PhD PROGRAM

QUALIFYING EXAM GUIDE

Qualifying exam provides an opportunity for candidates to consider a variety of topics related to faculty research areas. Candidates are expected to demonstrate their ability to bring a theoretical perspectives and research methodologies to bear on their intended area of research, thereby addressing their potential and readiness to proceed with independent thesis research in a PhD program where a commitment to a cross-disciplinary approach is a defining characteristic.

FOR PhD CANDIDATES IN MATHEMATICS

Each candidate must answer all questions (4 questions) in the compulsory part. Students are also expected to demonstrate their background in a chosen field of interest in the broadest sense by solving 4 questions from the selective part. In total, 8 questions among 19 should be answered.

The topics covered by the exam content and the distribution of questions to these topics can be seen from the following table.

	TOPICS	**# Questions**
COMPULSORY (for all students)	*Probability Theory and Mathematical Statistics*	2
	Advanced Mathematical Analysis	2
SELECTIVE (4 questions should be selected)	*Complex Analysis*	2
	Differential Equations (ODE and PDE)	3
	Linear Algebra	1
	Algebra	2
	Real Analysis	1
	Topology	2
	Functional Analysis	2
	Optimization	2
Total		19

Contents (Syllabi)

Advanced Mathematical Analysis

Applications of the Supremum/Infimum properties, Limit and Limit Theorems, Bolzano-Weierstrass Theorem, Cauchy Criterion, Convergent/Divergent Sequences/Series, Continuous Functions/Differentiation: Continuous Functions, Intermediate Value Theorem, Derivative, Rolle's Theorem, Mean Value Theorem, Uniform Continuity, Pointwise/Uniform Convergence of Sequences of Functions, Riemann Integrability, The Fundamental Theorem of Calculus, Applications of Integral, Double Integral, functions of bounded variation, The Riemann-Stieltjes integral, Multivariable functions and their properties.

References:

1. T. Apostol, Mathematical Analysis, 2nd Edition, Addison Wesley
2. R. G.. Bartle, D. R. Sherbert, "Introduction to Real Analysis", Wiley.

Probability Theory and Mathematical Statistics

Axioms of probability, Random variables, Distribution function, Discrete and continuous probability distributions, Expected value, Conditional expectation, Limit theorems, (Central limit theorem, Law of large numbers), Random sample and its properties, Principles of data reduction, Point estimation, Methods of finding point estimators, Methods of evaluating point estimators

References:

1. Probability theory by A.Borovkov,
2. Mathematical statistics by A.Borovkov,
3. The First Course in Probability by Sheldon Ross, Prentics Hall,
4. Statistical Inference by G. Casella, R. L. Berger, Duxbury Press, California

Topology

Cartesian products, relation, rule of assignment, function, image, inverse image, basic properties of set theory, Heine Borel theorem, Bolzano-Weierstrass theorem. Topology (Order, product, metric etc), interior, closure, boundary, limit points, isolated points, basis, subbasis, continuity and topological equivalence (homeomorphisms), subspace topology, Hausdorff spaces, T-1 axiom, countability and separation axioms. Connected and disconnected spaces, theorems of connectedness, path connectedness, locally connected and locally path connected spaces. Compact spaces and subspaces, compactness and continuity, properties related to compactness, one-point compactification, limit point compactness, sequentially compactness, Baire category theorem.

References:

1. J. R. Munkres, "Topology", Pearson Prentice Hall.
2. Fred H. Croom, Principles of Topology, The Saunders Series, ISBN 0-03-012813-7

Functional Analysis

Metric spaces, convergence, uniform convergence, Cauchy sequence, completeness, completion of metric spaces, Definitions of Normed and Banach spaces, properties of finite dimensional Normed spaces and subspaces, linear/bounded operators, continuity, functionals, normed spaces of operators, dual spaces, Definitions and properties of inner product and Hilbert spaces, relationship between Hilbert, Banach and complete metric spaces, inequalities (Hölder, Minkowski, Jensen, Schwartz etc) on normed spaces, orthogonality, orthonormal sets and sequences (Bessel inequality), series related to orthonormal sets and sequences (Fourier series), Legendre, Hermite and Laguerre polynomials, representation of functionals on Hilbert spaces, Hilbert-adjoint operator, Selfadjoint, unitary and normal operators, Hahn-Banach theorem, reflexive spaces, Category theorem, uniform boundedness theorem, strong and weak convergence, convergence of sequences of operators and functionals, open mapping theorem, closed linear operators closed graph theorem, Banach-fixed point theorem.

References:

1. E. Kreyszig, "Introductory Functional Analysis with Applications", Wiley.

Algebra

Groups, Homomorphisms, Isomorphism Theorems, Cayley's Theorem, Lagrange's Theorem, Permutation Groups, Fundamental Theorem of Abelian Groups, Rings, Ideals, Integral Domains, Euclidean Domains, Ring of Polynomials, Prime and Irreducible Elements, Principal Ideal Domains, Unique Factorization Domain, Fields, Field of Quotients, Field Extensions, Algebraic Closure, Finite Fields.

References:

1. Fraleigh, Abstract Algebra, Addison-Wesley, 2003.
2. M. Artin, Algebra, Prentice Hall, 1991.
3. I. N. Herstein, Topics in Algebra, Wiley, 1975

Linear Algebra

Vector Spaces, Bases, Dimension, Dual Spaces, Direct Sum, Transformations, Isomorphisms, Representations, Linear Functionals, Lagrange Interpolation, Determinants and Properties, Uniqueness, Canonical Forms, Eigenvalues, Eigenvectors, Invariant Subspaces, Simultaneous Triangulation and Diagonalization, Cyclic Decomposition Theorem, Cayley-Hamilton Theorem, Jordan Canonical Form. Inner Product Spaces, Orthogonal Projection, Adjoint Operators, Orthogonal Matrices, Hermitian Matrices, Sesqui-linear Forms, Positive Forms, Spectral Resolution, Polar Decomposition, Bilinear Forms, Quadratic Forms, Positive and Negative Definite Forms, Signature, Skew Symmetric Forms.

References:

1. K. Hoffman and R. Kunze, Linear Algebra, Prentice Hall, 1971.

Complex Analysis

Algebra of complex numbers: Algebraic operations, polar representations, roots of complex numbers.

Complex Calculus: Functions of a complex variable, elementary functions, limit, continuity, uniform continuity, derivative, Cauchy-Riemann equations, analytic functions, line integrals, Cauchy-Goursat theorem, residue theorems, Cauchy estimate theorem, Liouville's theorem, Morera's theorem, fundamental theorem of algebra, applications of residues for real improper integrals, maximum modulus principle, integration around branch cuts, Power, Taylor, Laurent series, uniform convergence, analytic continuation.

Geometry in the complex plane: Stereographic projection, linear fractional transformations, conformal mapping, mappings by other elementary functions.

References:

1. R. V. Churchill, J. W. Brown, Complex Variables and Applications.
2. L. Ahlfors, Complex Analysis.

Differential Equations

Classification of Differential Equations. Exact-Non Exact Differential Equations, Separable Equations, Linear Equations and Bernoulli Equations, The Homogeneous Linear Equation with Constant Coefficients, The Method of Undetermined Coefficients, Variation of Parameters, The Cauchy-Euler Equation, Power series solutions about an Ordinary Point, The Method of Frobenius, Bessel's Equation and Bessel Functions, Operator Method, The Matrix Method for Homogeneous Linear Systems with constant coefficients, Laplace Transform Solution of Linear Differential Equations with Constant Coefficients, Laplace Transform Solution of Linear Differential Equations with Discontinuous Non-homogeneous Terms, Picard's Theorem.

References:

1. Shepley L.Ross, "Introduction to Ordinary Differential Equations".

2. Nagle, Saff,Snider, “Fundamentals of Differential Equations and Boundary Value Problems”.

Partial Differential Equations

First Order Equations: Linear, Quasilinear, nonlinear equations. Classification of second order partial differential equations: Reduction to hyperbolic, parabolic, elliptic equations. Cauchy problem: for nonlinear first order equations, for linear second order equations, adjoint operators. Laplace, Poisson equations: Properties of harmonic functions. Separation of variables: Initial Boundary Value problem for Heat and Wave equations. Sturm-Liouville Problem and generalized Fourier series. Partial differential equations in polar, cylindrical and spherical coordinates. Green's Function: Solutions of boundary value problems or eigenvalue problems, Dirac δ-function, Green's function. D'Alembert's Method. Duhamel's Principle. The Fourier Transform and Its Applications: The Fourier Transform Method. The Heat Equation and Gauss's Kernel. The Fourier Cosine and Sine Transforms.

References:

1. R. Dennemeyer, “An introduction to partial differential equations and boundary value problems”.
2. N. H. Asmar, “Partial differential equations with Fourier series and Boundary value problems”.

Optimization

Convex sets, cones, convex hulls, convex functions, subgradients of convex functions, polyhedral sets, extreme points and extreme directions, linear programming and the simplex method, duality, Kuhn Tucker optimality conditions, Lagrange multipliers.

References:

1. M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali, Linear Programming and Network Flows, John Wiley, 2nd. Ed., New York (1990)
2. G.. B. Dantzig, Linear Programming and extensions, Princeton University Press, Princeton, NJ, (1963)

Real Analysis

Well ordering principle and countable ordinals, Algebra, sigma algebras, Measurable sets, Lebesgue measure, Measurable functions, Littlewood's three principles, Lebesgue Integral: The Lebesgue Integral, Bounded Convergence Theorem, Fatou's Lemma, Monotone Convergence Theorem, Lebesgue Dominated Convergence Theorem, Convergence in Measure, Differentiation and Integration: Vitali Lemma, Functions of Bounded Variation, Absolute Continuity, Convex Functions.

References:

1. H. L. Royden, Real Analysis Third Edition, Macmillan Publishing Company.

FOR PhD CANDIDATES IN STATISTICS

Each candidate must answer all questions (4 questions) in the compulsory part. Students are also expected to demonstrate their background in a chosen field of interest in the broadest sense by solving 4 questions from the selective part. In total, 8 questions among 19 should be answered.

The topics covered by the exam content and the distribution of questions to these topics can be seen from the following table.

	TOPICS	**# Questions**
COMPULSORY (for all students)	*Probability Theory and Mathematical Statistics*	2
	Advanced Mathematical Analysis	2
SELECTIVE (4 questions should be selected)	*Advanced Probability Theory*	5
	Statistical Inference	5
	Stochastic Processes	5
Total		19

Contents

Advanced Mathematical Analysis

Applications of the Supremum/Infimum properties, Limit and Limit Theorems, Bolzano-Weierstrass Theorem, Cauchy Criterion, Convergent/Divergent Sequences/Series, Continuous Functions/Differentiation: Continuous Functions, Intermediate Value Theorem, Derivative, Rolle's Theorem, Mean Value Theorem, Uniform Continuity, Pointwise/Uniform Convergence of Sequences of Functions, Riemann Integrability, The Fundamental Theorem of Calculus, Applications of Integral, Double Integral, functions of bounded variation, The Riemann-Stieltjes integral, Multivariable functions and their properties.

References:

T. Apostol, Mathematical Analysis, 2^{nd} Edition, Addison Wesley
R. G.. Bartle, D. R. Sherbert, "Introduction to Real Analysis", Wiley.

Probability Theory and Mathematical Statistics

Axioms of probability, Random variables, Distribution function, Discrete and continuous probability distributions, Expected value, Conditional expectation, Limit theorems, (Central limit theorem, Law of large numbers), Random sample and its properties, Principles of data reduction, Point estimation, Methods of finding point estimators, Methods of evaluating point estimators

References:

Probability theory by A.Borovkov,
Mathematical statistics by A.Borovkov,
The First Course in Probability by Sheldon Ross, Prentics Hall,
Statistical Inference by G. Casella, R. L. Berger, Duxbury Press, California

Advanced Probability Theory

Bivariate-multivariate distributions and their properties, Copulas and their properties, Order statistics (Distribution theory), Empirical distribution function and its properties, Glivenko-Cantelli theorem, Martingales

References:

Probability theory by A.Borovkov
Order Statistics by H.A. David and H.N. Nagaraja
An Introduction to Copulas by R.B. Nelsen

Statistical Inference

Random sample and its properties, Principles of data reduction, Point estimation, Methods of finding point estimators, Methods of evaluating point estimators, Hypothesis testing, Methods of finding tests, Methods of evaluating tests, Interval estimation, Bayesian approach, Linear regression, Nonparametric statistical techniques (One and two sample problems)

References:

Statistical Inference by G. Casella, R. L. Berger, Duxbury Press, California
Mathematical Statistics by J. Shao
Nonparametric Statistical Inference by Gibbons and Chakraborti

Stochastic Processes

Definition and characteristics of stochastic processes, Poisson process, Renewal process, Markov processes, Markov chains, First step analysis of Markov chains, Kolmogorov-Chapman equations, Birth-Death Processes, Stationary processes, Branching processes, Queuing systems

References:

Stochastic Processes by S. Ross
An Introduction to Stochastic Modeling by H. Taylor and S. Karlin